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Fuzzy distance transform: theory, algorithms, and applications

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Authors
[Punam K.](#)

Medical Image Processing Group, Department of Radiology, University of Pennsylvania, Fourth Floor,

[Saha](#)

Blockley Hall, 423 Guardian Drive, Philadelphia, Pennsylvania

[Felix W. Wehrli](#)

Laboratory for Structural NMR, Department of Radiology, University of Pennsylvania Health System, 3400

Spruce Street, Philadelphia, Pennsylvania

[Bryon R.](#)

Department of Bio-engineering, University of Pennsylvania Health System, 3400 Spruce Street, Philadelphia,

[Gomberg](#)

Pennsylvania

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↑ ABSTRACT

This paper describes the theory and algorithms of distance transform for fuzzy subsets, called fuzzy distance transform (FDT). The notion of fuzzy distance is formulated by first defining the length of a path on a fuzzy subset and then finding the infimum of the lengths of all paths between two points. The length of a path n in a fuzzy subset of the n -dimensional continuous space R^n is defined as the integral of fuzzy membership values along n . Generally, there are infinitely many paths between any two points in a fuzzy subset and it is shown that the shortest one may not exist. The fuzzy distance between two points is defined as the infimum of the lengths of all paths between them. It is demonstrated that, unlike in hard convex sets, the shortest path (when it exists) between two points in a fuzzy convex subset is not necessarily a straight line segment. For any positive number $\theta \leq 1$, the θ -support of a fuzzy subset is the set of all points in R^n with membership values greater than or equal to θ . It is shown that, for any fuzzy subset, for any nonzero $\theta \leq 1$, fuzzy distance is a metric for the interior of its θ -support. It is also shown that, for any smooth fuzzy subset, fuzzy distance is a metric for the interior of its 0-support (referred to as *support*). FDT is defined as a process on a fuzzy subset that assigns to a point its fuzzy distance from the complement of the support. The theoretical framework of FDT in continuous space is extended to digital cubic spaces and it is shown that for any fuzzy digital object, fuzzy distance is a metric for the support of the object. A dynamic programming-based algorithm is presented for computing FDT of a fuzzy digital object. It is shown that the algorithm terminates in a finite number of steps and when it does so, it correctly computes FDT. Several potential applications of fuzzy distance transform in medical imaging are presented. Among these are the quantification of blood vessels and trabecular bone thickness in the regime of limited special resolution where these objects become fuzzy.

↑ REFERENCES

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- 1 1. A. Rosenfeld and J. Pfaltz, Distance functions in digital pictures, *Pattern Recog.* 1, 1968, 33-61.
- 2 2. P. E. Danielsson, Euclidean distance mapping, *Comput. Graphics Image Process.* 14, 1980, 227-248.
- 3 3. G. Borgefors, Distance transformations in arbitrary dimensions, *Comput. Vision Graphics Image Process.* 27, 1984, 321-345.
- 4 Gunilla Borgefors, On digital distance transforms in three dimensions, *Computer Vision and Image Understanding*, v.64 n.3, p.368-376, Nov. 1996 [doi>[10.1006/cviu.1996.0065](https://doi.org/10.1006/cviu.1996.0065)]
- 5 5. A. Kaufmann, *Introduction to the Theory of Fuzzy Subsets*, Vol. 1, Academic Press, New York, 1975.
- 6 6. A. Rosenfeld, Fuzzy digital topology, *Inform. Control* 40, 1979, 76-87.
- 7 7. J. C. Bezdek and S. K. Pal, *Fuzzy Models for Pattern Recognition*, IEEE Press, New York, 1992.
- 8 8. K. S. Fu and A. Rosenfeld, Pattern recognition and image processing, *IEEE Trans. Comput.* 25, 1976, 1336-1346.
- 9 Stephen M. Pizer, David Eberly, Daniel S. Fritsch, Zoom-invariant vision of figural shape: the mathematics of cores, *Computer Vision and Image Understanding*, v.69 n.1, p.55-71, Jan. 1998 [doi>[10.1006/cviu.1997.0563](https://doi.org/10.1006/cviu.1997.0563)]
- 10 Punam K. Saha, Jayaram K. Udupa, Dewey Odhner, Scale-based fuzzy connected image segmentation: theory, algorithms, and validation, *Computer Vision and Image Understanding*, v.77 n.9, p.145-174, Feb. 2000
- 11 11. S. N. Srihari and J. K. Udupa, Understanding the bin of parts, in *Proceedings of International Conference on Cybernetics and Society*, Denver, Colorado, 1979, pp. 44-49.
- 12 12. Y. Tsao and K. S. Fu, A parallel thinning algorithm for 3D pictures. *Comput. Graphics Image Process.* 17, 1981, 315-331.
- 13 13. P. K. Saha, B. B. Chaudhuri, and D. Dutta Majumber, A new shape preserving parallel thinning algorithm for 3D digital images, *Pattern Recog.* 30, 1997, 1939-1955.
- 14 Jean Serra, *Image Analysis and Mathematical Morphology*, Academic Press, Inc., Orlando, FL, 1983
- 15 15. G. Borgefors, Applications of distance transformations, in *Aspects of Visual Form Processing* (C. Arcelli et al., Eds.), pp. 83-108, World Scientific, Singapore, 1994.
-  16 Azriel Rosenfeld, John L. Pfaltz, Sequential Operations in Digital Picture Processing, *Journal of the ACM (JACM)*, v.13 n.4, p.471-494, Oct. 1966 [doi>[10.1145/321356.321357](https://doi.org/10.1145/321356.321357)]
- 17 17. N. Okabe, J. Toriwaki, and T. Fukumura, Paths and distance functions in three dimensional digitized pictures, *Pattern Recog. Lett.* 1, 1983, 205-212.
- 18 Ingemar Ragnemalm, The Euclidean distance transform in arbitrary dimensions, *Pattern Recognition Letters*, v.14 n.11, p.883-888, Nov. 1993 [doi>[10.1016/0167-8655\(93\)90152-4](https://doi.org/10.1016/0167-8655(93)90152-4)]

- 19 Gunilla Borgefors, Distance transformations in digital images, Computer Vision, Graphics, and Image Processing, v.34 n.3, p.344-371, June 1986 [doi>[10.1016/S0734-189X\(86\)80047-0](https://doi.org/10.1016/S0734-189X(86)80047-0)]
- 20 Ben J. H. Verwer, Local distances for distance transformations in two and three dimensions, Pattern Recognition Letters, v.12 n.11, p.671-682, Nov. 1991 [doi>[10.1016/0167-8655\(91\)90004-6](https://doi.org/10.1016/0167-8655(91)90004-6)]
- 21 A. L. D. Beckers , A. W. M. Smeulders, Optimization of length measurements for isotropic distance transformations in three dimension, CVGIP: Image Understanding, v.55 n.3, p.296-306, May 1992 [doi>[10.1016/1049-9660\(92\)90027-Z](https://doi.org/10.1016/1049-9660(92)90027-Z)]
- 22 22. D. Rutovitz, Data structures for operations on digital images, in Pictorial Pattern Recognition (G. C. Cheng et al., Eds.), pp. 105-133, Thompson, Washington, DC, 1968.
- 23 23. G. Levi and U. Montanari, A gray-weighted skeleton, Inform. and Control 17, 1970, 62-91.
- 24 Azriel Rosenfeld , Avinash C. Kak, Digital Picture Processing, Academic Press, Inc., Orlando, FL, 1982
- 25 25. J. Mordeson and P. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Physica-Verlag, Berlin, 2000.
- 26 26. E.W. Weisstein, CRC Concise Encyclopedia of Mathematics, Chapman & Hall/CRC, Boca Raton, FL, 1999.
- 27 27. A. Rosenfeld, The diameter of a fuzzy set, Fuzzy Sets Systems 13, 1984, 241-246.
- 28 A. Bogomolny, On the perimeter and area of fuzzy sets, Fuzzy Sets and Systems, v.23 n.2, p.257-269, Aug. 1987 [doi>[10.1016/0165-0114\(87\)90062-5](https://doi.org/10.1016/0165-0114(87)90062-5)]
- 29 29. A. Rosenfeld, The fuzzy geometry of image subsets, Pattern Recog. Lett. 2, 1991, 311-317.
- 30 Jayaram K. Udupa , Supun Samarasekera, Fuzzy connectedness and object definition: theory, algorithms, and applications in image segmentation, Graphical Models and Image Processing, v.58 n.3, p.246-261, May 1996 [doi>[10.1006/gmip.1996.0021](https://doi.org/10.1006/gmip.1996.0021)]
- 31 31. Z. H. Cho, J. P. Jones, and M. Sing, Foundations of Medical Imaging, Wiley, New York, 1993.
- 32 32. N. R. Pal and S. K. Pal, A review of image segmentation techniques, Pattern Recog. 26, 1993, 1277-1294.
- 33 Jayaram K. Udupa , Gabor T. Herman, 3D imaging in medicine, CRC Press, Inc., Boca Raton, FL, 1991
- 34 34. M. Sonka, V. Hlavac, and R. Boyle, Image Processing, Analysis, and Machine Vision, 2nd ed., PWS Publishing, Brooks/Cole, Pacific Grove, CA, 1999.
- 35 35. T.Y. Kong, A. W. Roscoe, and A. Rosenfeld, Concepts of digital topology, Topology Appl. 46, 1992, 219-262.
- 36 36. P. K. Saha, J. K. Udupa, and J. M. Abrahams, Automatic bone-free rendering of cerebral aneurysms via 3D-CTA, in Proceedings of SPIE: Medical Imaging, San Diego, CA, Vol. 4322, pp. 1264-1272, 2001.
- 37 37. S.N. Hwang and F. W. Wehrli, Estimating voxel volume fraction of trabecular bone on the basis of magnetic resonance images acquired in vivo, Internat. J. Imaging Systems Tech. 10, 1999, 186-198.

- 38 38. Z. Wu, H. Chung, and F. W. Wehrli, A Bayesian approach to subvoxel tissue classification in NMR microscopic images of trabecular bone, *Magnetic Res. Med.* 31, 1994, 302-308.
- 39 39. J. E. Aaron, N. B. Makins, and K. Sagreiya, The microanatomy of trabecular bone loss in normal aging men and women, *Clinical Orthopaedics Related Res.* 215, 1987, 260-271.
- 40 Jayaram K. Udupa , Dewey Odhner, Shell Rendering, IEEE Computer Graphics and Applications, v.13 n.6, p.58-67, November 1993 [doi>10.1109/38.252558]
- 41 41. J. K. Udupa, D. Odhner, S. Samarasekera, R. J. Goncalves, K. Iyer, K. Venugopal, and S. Furuie, 3DVIEWNIX: A open, transportable, multidimensional, multimodality, multiparametric imaging system, *Proc. SPIE2164*, 1994, 58-73.

↑ CITED BY 2

Stina Svensson, A decomposition scheme for 3D fuzzy objects based on fuzzy distance information, Pattern Recognition Letters, v.28 n.2, p.224-232, January, 2007

Leena Ikonen , Pekka Toivanen, Distance and nearest neighbor transforms on gray-level surfaces, Pattern Recognition Letters, v.28 n.5, p.604-612, April, 2007

↑ INDEX TERMS

Primary Classification:

I. Computing Methodologies

↳ **I.4 IMAGE PROCESSING AND COMPUTER VISION**

↳ **I.4.5 Reconstruction**

↳ **Subjects: Transform methods**

Additional Classification:

I. Computing Methodologies

↳ **I.2 ARTIFICIAL INTELLIGENCE**

↳ **I.2.10 Vision and Scene Understanding**

↳ **Subjects: Shape**

↳ **I.2.8 Problem Solving, Control Methods, and Search**

↳ **Subjects: Dynamic programming**

↳ **I.5 PATTERN RECOGNITION**

↳ **I.5.1 Models**

↳ **Subjects: Fuzzy set**

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Algorithms, Experimentation, Theory

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adjacency, distance transform, dynamic programming, fuzzy distance, fuzzy subset, metric, path

↑ Collaborative Colleagues:

Bryon R. Gomberg: Punam K. Saha

Punam K. Saha: Felix W. Wehrli
Bryon R. Gomberg
Roberto A. Lotufo
D. Dutta Majumder
Dewey Odhner
Azriel Rosenfeld
Jayaram K. Udupa
Felix W. Wehrli
Ying Zhuge

Felix W. Wehrli: Bryon R. Gomberg
Scott N. Hwang
Scott Nyshin Hwang
Punam K. Saha
Hee Kwon Song

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